Dynamic Model of Magnetic Amplifier

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Abstract

Magnetic amplifiers, although having heavy weight indicators, and also relatively low amplification coefficient are still largely used in electrical drive control systems. This is due to their high reliability and simple operation. A magnetic amplifier can be considered as a nonlinear controlled inductance, whose reactive conductance depends on controlled current. In the paper we propose a dynamic model of magnetic amplifier.

Keywords: *Dynamic Model, Magnetic Amplifier.*

Introduction

Magnetic amplifiers came into being when the last century was just one year old. It would be six years - in 1907 - before a youngster named Lee de Forest would make news with his audion, the world's first vacuum-tube amplifier. And the transistor was still 47 years in the future. For a while, it looked as though the magnetic amplifier would hold its own against that upstart, the audion. In 1910 F.W. Alexanderson, the electronic pioneer, employed magnetic amplifiers to modulate his early transmitters and World War I transmitting stations used his circuits. By the early 20's, however the flashy vacuum tube had taken and the "magnetic" was almost forgotten in the country. It was not forgotten in Germany, however, as we found out when World War II started. In the years between the wars, the Germans had brought the magnetic amplifier to a high state of development. War-time found them using magnetics for reliable, accurate, and trouble-free control of everything from gun turrects to automatic pilot systems, and they even used them in the V2 rockets (Mullett and Charles, 1984:922).

Awakened to the possibilities inherent in this design, Allied scientists began to push the development of magnetic amplifiers. Before much progress had been made, though, the war was over. But the spark had been kindled, a few years later Vickers Inc. (now a division of Sperry Rand) came out with the first commercially produced magnetics (Venable and H. Dean, 1983:10).

By that time, interest had been aroused all over the world. In the following decade, hundreds of other firms, including all the big names in electrical and electronic equipment, have added magnetics to their product lines. And almost no branch of industry now operates without them.

Construction of Magnetic Amplifier

Magnetic amplifier is built in the form of two steel magnetoconductors A and B with windings (fig1). Magnetoconductor of magnetic amplifier is built with ferromagnetic material. The dependence of magnetic induction B on magnetic field H for the ferromagnetic material is called magnetic curve, it is nonlinear (Venable and H. Dean, 1983:10).

Without consideration of hysteresis curse we can make an approximation function:

$$B = B_b.\arctan\left(\frac{H}{H_b}\right) \tag{1}$$

Where B_b, H_b – parameters whose values depend on the type of ferromagnetic material.

The quotient $\mu_b = B_b/H_b$ is the absolute permeability of ferromagnetic material.

In figure 2 we represent magnetic curve of electrotechnical steel

$B = 0{,}96.\arctan(\frac{H}{75})$, and also magnetic curve of a type of alloy $B = 0{,}7.\arctan(\frac{H}{2})$.
In the magnetoconductors, we enrol functioning windings and control winding (fig.1).

Functioning windings are separately enrolled in each magnetoconductor and they have equal number of turns W_1. Functioning windings are joined specifically.

Control winding is common for both magnetoconductors and it has a number of turns W_2.

Main Fluxes of Magnetoconductors and Main Total Fluxes of Windings in Magnetic Amplifier

The parameters of windings $K = 1{,}2$ are often characterized by active resistances and inductances. We mention that in magnetic amplifier, inductances are not constant expressions and depend on currents of functioning windings i_1 and control winding i_2 (C.V. Aggers and W. E. Pakala, 1937:55).

Currents i_k (k = 1,2) that circulate along functioning windings and control winding create magnetomotrice forces $F_k = w_k \times i_k$, on whose action appear magnetic fluxes.

Magnetic fluxes are divided into dispersion fluxes and main fluxes. Dispersion fluxes of winding do not interact with other windings. The relation between dispersion flux linkage ψ_k and winding current i_k is considered linear:

$\psi_k = L_k \times i_k$, where L_k is dispersion inductance ; k = 1,2

The main magnetic fluxes linking up with other windings and circulate along magnetoconductors. They are nonlinear functions of currents that create them.

We will consider that magnetic fields are distributed along sections of magnetoconductors A and B uniformly and they are defined according to total current law:

$$H_A = (F_1 + F_2)/l = (w_1 i_1 + w_2 i_2)/l\,;$$
$$H_B = (F_1 - F_2)/l = (w_1 i_1 - w_2 i_2)/l$$

Where l – the middle length magnetoconductor magnetic force line.

$F_1 = w_1 i_1$, $F_2 = w_2 i_2$ – magnetomotrice forces of functionnning and control windings.

Magnetic induction in magnetoconductors are determined by equation (1):

$B_A = B_b \arctan(H_A/H_b)$; $B_B = B_b \arctan(H_B/H_b)$.

The main magnetic fluxes of magnetoconductors :

$$\Phi_{AA} = S \times B_A = S \times B_b \arctan[(w_1 i_1 + w_2 i_2)/(H_b \times l)]\,; \quad (2)$$
$$\Phi_{BB} = S \times B_B = S \times B_b \arctan[(w_1 i_1 - w_2 i_2)/(H_b \times l)]\,;$$

Where S – section of magentoconductor.

The main flux linkages of functioning windings are related with magnetic fluxes (2) :
$$\psi_{1_{1_A}}(i_1, i_2) = w_1 \Phi_{AA}\,;\ \psi_{1_{1_B}}(i_1, i_2) = w_1 \Phi_{BB} \quad (3)$$

Since functioning windings are connected in series and are located in different magnetoconductors, then main flux linkage of functioning windings.

$$\psi_{1_1}(i_1, i_2) = w_1(\Phi_{AA} + \Phi_{BB}) \quad (4)$$

The main flux linkage of control winding is related with magnetic fluxes (2) by equation:

$$\psi_{2_2}(i_1, i_2) = w_2(\Phi_{AA} - \Phi_{BB}) \quad (5)$$

Dynamics of Electromagnetic Processes in Functioning Windings of Magnetic Amplifier

When circulation along windings of magnetic amplifier occurs by i_1 and i_2, in serial connected functioning windings there will be voltage (C.V. Aggers and W. E. Pakala, 1937:55).

$$U_1 = U_{1_A} + U_{1_B} = 2R_1 i_1 + 2L_1 P \cdot i_1 + P\psi_{1_1}(i_1, i_2) \quad (6)$$

Where R_1 – active resistance of functioning winding;

L_1 – dispersion inductance of functioning winding.
P – operator of differentiation
$\psi_{11}(i_1, i_2)$ – main total flux linkage of functioning windings.

It is obvious that equation (6) is nonlinear because $\psi_{11}(i_1, i_2)$ is a nonlinear function of i_1, i_2.
In per-unit system we have the equation

$$\psi^*_{11}(i^*_1, i^*_2) = \arctan(i^*_1 + i^*_2) + \arctan(i^*_1 - i^*_2) \in [0, \pi]$$
We suppose that the voltage in functioning winding u_1 is a periodic function with angle speed ω. Obviously the solution of the latter equation will also be periodic with angle speed ω.

For evaluation of dynamic properties of functioning winding, the free component of solution could be approximated as:

$$i_{1f} = \exp\left(-\frac{t}{T_{11}}\right)\sin(\omega t) \quad (7)$$

Where T_{11} – time constant related to functioning winding.

We write characteristic equation for differential equation (6) for first harmonic, using relation (7).

For that purpose, we replace free component in the right part of (6) and equalizing to zero. We find time constant of functioning winding:

$$T_{11} = \frac{L_{b1}}{(1+i_2^{*2}).R_1} + \frac{L_1}{R_1}$$

Where $L_{b1} = \frac{\psi_{b1}}{I_{b1}} = W_1^2.S.B_b/(H_b.l)$

The voltage created in functioning winding has components from dispersion fluxes and main magnetic flux. The voltage from main magnetic flux:

$u_{11}^* = p\psi_{11}(i_1^*, i_2^*)$ is considered as main voltage. Its expression depends both on functioning current and on control current.

We will assume that along control winding circulates constant current i_2, while along functioning windings connected in serie sinusoidal current

$$i_1 = I_{m1}\sin(t^*)$$

Where $t^* = \omega t$.

With circulation along functioning winding of sinusoidal current, there is creation of non-sinusoidal voltage. Non sinusoidal components on voltage is caused by nonlinear dependence of main flux linkage $\psi_{11}(i_1, i_2)$ of currents i_1 and i_2.

The main voltage is functioning winding of magnetic amplifier is approximated by equivalent cosinusoid $U_{m11}\cos(\omega t)$. For that purpose we equalize in half period of current the integral of main voltage.

$\int_{-\pi/2}^{\pi/2} \omega \frac{d}{dt^*}\psi_{11}(I_{m1}.\sin(t^*), i_2)dt^* = 2\omega\psi_{11}(I_{m1}, i_2)$ and equivalent of cosinusoid

$$\int_{-\pi/2}^{\pi/2} U_{m11}.\cos(t^*)\,dt^* = 2U_{m11}$$

From equality of integrals, $U_{m11} = \omega\psi_{11}(I_{m1}, i_2)$.

The amplitude of the equivalent cosinusoid of main voltage, written in per units, with consideration of the relation $\psi_{11}^*(t_1^*, t_2^*) = \arctan(i_1^* + i_2^*) + \arctan(i_1^* - i_2^*)\epsilon[0, \pi]$ will look as follows:
$U_{m11}^*(I_{m1}^*, i_2^*) = \psi_{11}^*(I_{m1}^*, i_2^*) = \arctan(I_{m1}^* + i_2^*) + \arctan(I_{m1}^* - i_2^*)\epsilon[0, \pi]$
Apart from main voltage U_{m11}, in functioning windings also appear voltages drop in dispersion inductance $\omega L_1.I_{m1}$ and in active resistance $R_1 I_{m1}$. The latter is so small that it can be neglected. Thus the amplitude of voltage in functioning windings could be considered.

$$U_{m1} = \omega\psi_{11}(I_{m1}, i_2) + 2\omega L_1 I_{m1}.$$

The given expression is the Volt-Ampere characteristic of functioning winding of magnetic amplifier. In per unit system, it looks as follows:

$$U^*_{m1} = \arctan(I^*_{m1} + i^*_2) + \arctan(I^*_{m1} - i^*_2) + 2L^*_1 I^*_{m1} \quad (8)$$

Volt-ampere characteristics functioning windings for various control currents i^*_2, are constructed expression (8) presented in figure 3. These characteristics are similar to those of transistors.

According to these characteristics we could distinguish four functioning regimes of magnetic amplifier: cutting, electrical disruption, saturation and active regime (E.L. Harder, 1930:601). The *cutting regime* corresponds to the linear part of the volt-ampere characteristic with $i^*_2 = 0$. In this regime, functioning windings have maximal resistance:

$$X^*_0 = \left.\frac{dU^*_{m1}}{dI^*_{m1}}\right]_{i^*_2=0} = 2$$

In the cutting regime, the voltage in functioning windings belongs to the interval $U_{m1} \epsilon [0, U_{m1N}]$
Where U_{m1N} – amplitude of nominal voltage. The value of nominal voltage in per-units $U^*_{m1N} = 2{,}0, \dots, 2{,}5$.

The *electrical disruption regime* appears for $U^*_{m1} \geq \pi$. For the value of main voltage $U^*_{m1} = \pi$ we observe stabilization of functioning windings voltage U^*_{m1}, with suddent high current. The differential resistance of volt-ampere characteristic part corresponding to electrical disruption is determined by dispersion resistance ωL_1.

The *saturation* regime is characterized by the value of main voltage $U^*_{m1} = 0$. In this regime, minimal voltage is dropped in functioning windings. The differential resistance is then determined by dispersion resistance ωL_1.

Active regime corresponds to the part of volt-ampere characteristics with $i^*_2 > 0$, having vertical segment with almost constant value of functioning winding current.

We define the relation of current amplitude for equivalent functioning winding sinusoid depending on control current in active regime. We consider a given value U^*_{m1} that corresponds to an active regime.
For $i^*_2 = 0$, expression (8) looks as follows: $U^*_{m1} = 2\arctan(I^*_{m10}) + 2.L_1 I^*_{m10}$ (9)

Where I^*_{m10} is the amplitude of functioning winding equivalent sinusoid current for $i^*_2 = 0$.

From equations (8) and (9), when neglecting the dispersion inductance L1, we have the following relation:

$$I^*_{m1} = (I^{*2}_{m10} - 1 + \sqrt{(1 + I^{*2}_{m10})^2 + 4.i^{*2}_2.I^{*2}_{m10}})/(2.I_{m10})$$

The graphic of function of winding current I^*_{m1}depending on control current in active regime is represented in figure 4. We could make an approximation $I^*_{m1} \approx |i^*_2|$ or $I_{m1} \approx \beta|i_2|$, where $\beta = W_2/W_1$ is amplification coefficient of current.

Dynamic Characteristics of Control Winding

When there is circulation along windings of magnetic amplifier of currents i_2 and i_1, in control windings appears the voltage (A. Boyajian, 1924:925).

$$u_2 = R_2 . i_2 + L_2 p i_2 + p\psi_{22}(i_1, i_2) \quad (10)$$

Where R_2 – active resistance of control winding;

L_2 – dispersion inductance of control winding;
$\psi_{22}(i_1, i_2)$ – main total flux linkage of control winding.

Obviously (10) is nonlinear equation because of ψ_{22}. In per unit system, $\psi_{22}(i_1, i_2)$ has similar aspect as shown in figure 4.

We assume that along functioning windings is circulating a sinusoidal current $i_1 = I_{m1}\sin(\omega t)$. The total flux linkage will be a pulsation function of time with frequency 2ω (H.S. Kirschbaum, E.L. Harder 1947:275).

The flux linkage of control winding has the following minimal and maximal values respectively:

$$\psi^*_{min} = \arctan(I^*_{1m} + i^*_2) - \arctan(I^*_{1m} - i^*_2)\,;$$
$$\psi^*_{max} = 2\arctan(i^*_2)$$

The plot that illustrates the character of pulsations of control winding flux linkage with sinusoidal current of functioning winding is shown in figure 5.

The pulsation period of control winding flux linkage is considerably smaller than control winding time constant. Therefore we can neglect pulsation when building the control winding dynamic model. We consider that the medium value of control winding flux linkage.

$$\psi^*_m(i^*_2) \approx 3/2\arctan(i^*_2)$$

The plot is represented on figure (6).

The main inductance of control winding is:

$$L^*_{22} = \frac{d\psi^*_m(i^*_{i2})}{di^*_2} = \frac{3/2}{1 + i^{*2}_2}$$

The time constant of control winding:

$$T^*_{02} = \frac{3/2}{(1 + i^{*2}_2).R^*_2} + \frac{L^*_2}{R^*_2}$$

From the latter expression, it appears that for big current values regime in control winding the fast action of magnetic amplifier grows. In the electrical disruption and saturation regimes, the time constant is only determined by dispersion inductance.

Therefore, in the regime of high values of magnetic fields, functioning windings magnetic amplifier are current source whose value depends on current value in control winding.

Conclusion

The main functioning regime of magnetic amplifier is the active regime. This corresponds to the case when $i_2^* > 3$. The functioning windings behave as a controlled current source.

The amplitude of equivalent sinusoid of current and voltage in functioning windings consideration of energy losses is deviated by 90°. Magnetic amplifier is a nonlinear dynamic element. We can evaluate the time constant of magnetic amplifier as the sum of time constants of functioning and control windings. The fast action of magnetic amplifier considerably depends on control current i_2.

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Figures

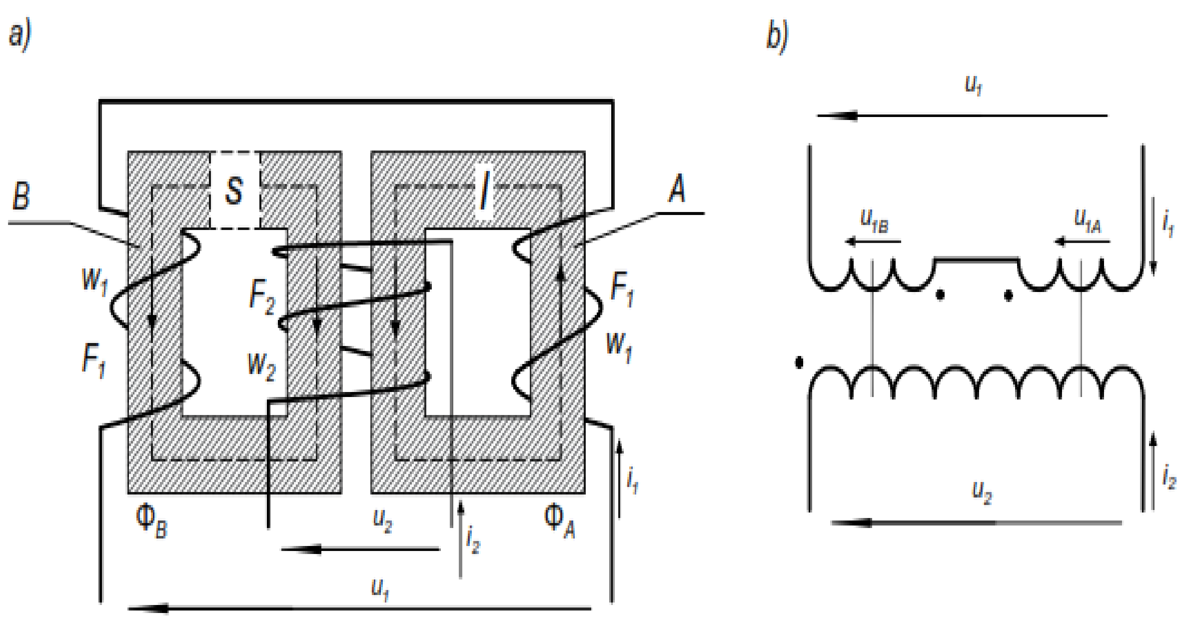

Figure 1 – Magnetic amplifier a) Construction b) basic circuit.

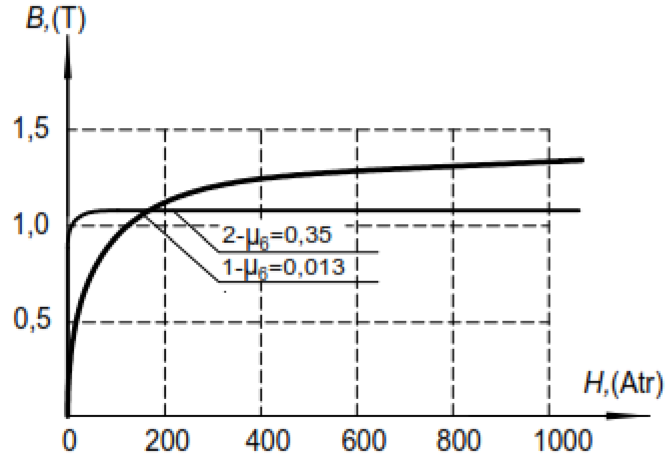

Figure 2 – Magnetic curves 1 – electrotechnical steel 2 – Alloy.

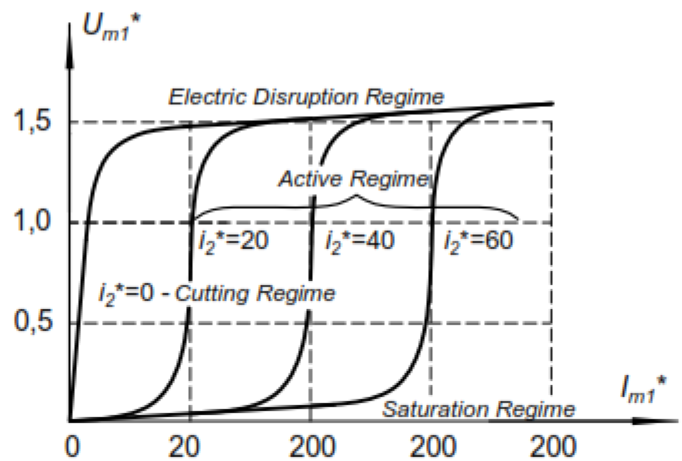

Figure 3 – The Set of Volt-Ampere characteristics functioning windings for various control currents i_2^*.

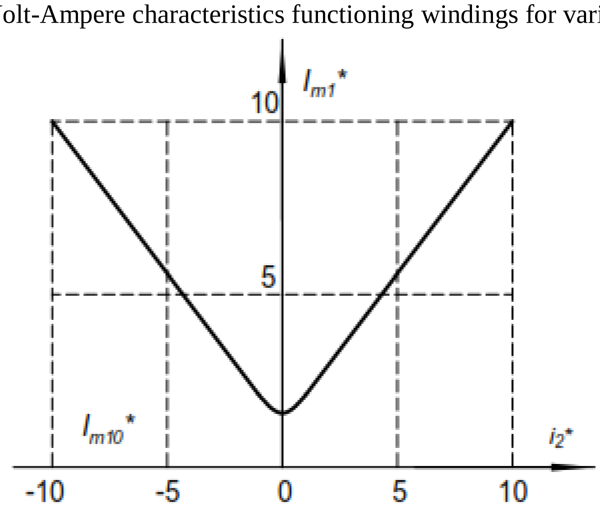

Figure 4 – Dependence of current I_{m1}^* on control current i_2^* in active regime.

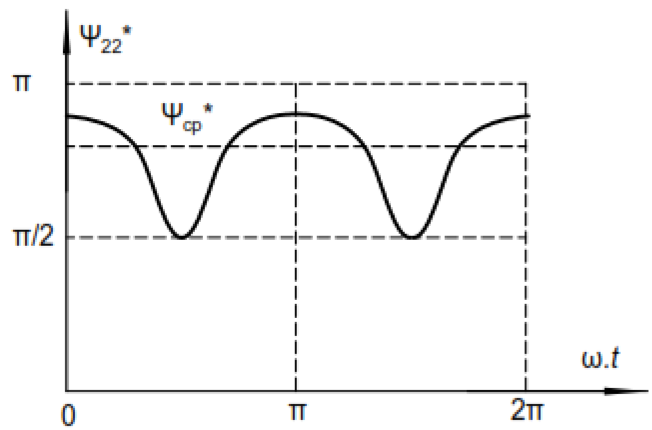

Figure 5 – Flux linkage pulsations of control winding with sinusoidal current of functional winding ($I^*_{m1} = 10$ and $i^*_2 = 10$).

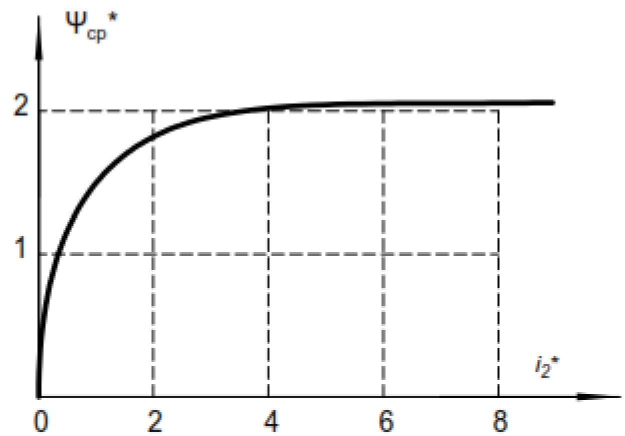

Figure 6 – Dependence of medium flux linkage of control winding on its current.